Sample Paper 2014: Paper 1

Question 8 (50 marks)

Formulae and Tables Book
Financial mathematics: Compound interest [page 30]

$$F = P(1 + i)^t$$

t = Time period (in years)
i = (Annual) rate of interest expressed as a decimal
F = Final value
P = Principal
Note: The time period can be months or weeks instead of years provided the interest rate is given for that time period.

Question 8 (a)

$P = ?$

$F = €20\ 000$

$t = 1$ year

$i = 0 \cdot 03$

$$P = \frac{F}{(1+i)^t} = \frac{20\ 000}{(1+0 \cdot 03)^1} = €19\ 417 \cdot 48$$

Question 8 (b)

$$P = \frac{20\ 000}{(1.03)^t}$$

Question 8 (c)

I need to calculate the retirement fund that he has saved for and is available on the day of his retirement. €20 000 is drawn down immediately. The next €20 000 will not be drawn down for another year so its present value on the day of retirement is $\frac{20\ 000}{(1.03)^1}$. The next €20 000 will not be drawn from the retirement fund for 2 years so its present value is $\frac{20\ 000}{(1.03)^2}$. An so on.

$$20\ 000 + \frac{20\ 000}{(1 \cdot 03)^1} + \frac{20\ 000}{(1 \cdot 03)^2} + ... + \frac{20\ 000}{(1 \cdot 03)^{24}}$$

$$= 20\ 000\left[1 + \frac{1}{1 \cdot 03} + \frac{1}{1 \cdot 03^2} + ... + \frac{1}{1 \cdot 03^{24}}\right]$$

$$a = 1,\ r = \frac{1}{1 \cdot 03},\ n = 25$$

$$S_n = 20\ 000\left[\frac{1(1-(\frac{1}{1 \cdot 03})^{25})}{1-\frac{1}{1 \cdot 03}}\right] = €358\ 710 \cdot 84$$

Formulae and Tables Book
Sequences and series:
Geometric series [page 22]

$$S_n = \frac{a(1-r^n)}{1-r}$$

Question 8 (d)

(i) $(1+i)^{12} = 1.03$

$1+i = 1.03^{\frac{1}{12}} = 1.002466$

$\therefore i = 0.002466 = 0.2466\%$

(ii) $F = P(1.002466)^n$

His first payment P will be compounded 480 times at an interest rate of 0.2466%. His second payment P will be compounded 479 times at an interest rate of 0.2466%. And so on.

(iii) $358\ 710 \cdot 84 = P(1 \cdot 002466)^{480} + ... + P(1 \cdot 002466)^1$

$$= \frac{P(1 \cdot 002466)(1-(1 \cdot 002466)^{480})}{1-1 \cdot 002466}$$

$\therefore P = €390 \cdot 17$

Question 8 (e)

10 years less: $30 \times 12 = 360$ months

$$358\ 710 \cdot 84 = \frac{P(1 \cdot 002466)(1-(1 \cdot 002466)^{360})}{1-1 \cdot 002466}$$

$\therefore P = €618 \cdot 35$